Remainders and quotients

This module is about integers (whole numbers), dividing, quotients and remainders. It is useful when programming and using type `int` (short for integer). For example, few people would say that 17 days is $2\frac{3}{7}$ weeks. Instead we stay with integers and divide 17 by 7. We find that 17 days is 2 weeks and 3 days.
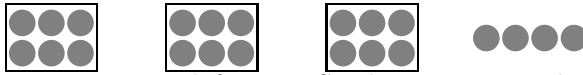
Here is an example.

Example 1.

I'm packing eggs into boxes which hold 6 eggs each. I have 22 eggs.

How many boxes can I fill and how many eggs do I have left over?

Answer. Using 3 boxes I can pack $3 \times 6 = 18$ eggs.

I have $22 - 18 = 4$ eggs left over. So the answer is 3 boxes with 4 eggs left over.

In this context I would not say that I'd packed $3\frac{4}{6}$ or $3\frac{2}{3}$ boxes.

Here are some similar questions.

Example 2.

How many years and months is 18 months?

Answer. It is 1 year and 6 months. We could say it's 1.5 years, but we want a whole number of years together with the number of months left over.

Example 3. How many minutes and seconds is 134 seconds?

Answer. Each minute is 60 seconds, and 134 is $2 \times 60 + 14$, so it's 2 minutes and 14 seconds.

We would not say that it's 1 minute and 74 seconds, as 74 seconds is more than a minute.

Example 4. How many weeks and days is 25 days?

Answer. It's 3 weeks and 4 days because $25 = 7 \times 3 + 4$.

In each of these questions we have taken a number, divided it by another number to get the *quotient* and then worked out how much was left over, the *remainder*.

In Example 1. we divided 22 by 6. We found that 22 divided by 6 is 3 with remainder 4. The quotient is 3. We could also say "22 divided by 6 is 3 with 4 left over" or "6 goes into 22 3 times with remainder 4". If we were happy to work with fractions we could say "22 divided by 6 is $3\frac{4}{6}$", but that doesn't tell us directly how many boxes were filled and how many eggs were left.

In general, for integers (whole numbers) a and b with $b > 0$, we say that a divided by b is q with remainder r. This means that

$$a = b \times q + r,$$

q and r are integers with $0 \leq r < b$ (r is greater than or equal to zero, but less than b). We could also say

"b goes into a q times with remainder r"

"a divided by b is q with remainder r".

There is only one correct quotient and remainder. The remainder is always 0 or positive; it is also always less than b. In Example 1. we divided 22 by 6:

$$22 = 6 \times 3 + 4.$$

quotient   remainder

and so 22 divided by 6 is 3 with remainder 4. The quotient is 3 and the remainder is 4. In Example 2., 18 was divided by 12; the quotient is 1 and the remainder 6:

$$18 = 12 \times 1 + 6.$$

quotient   remainder

To determine whether a number is even or odd, look at the remainder after it has been divided by 2: an even number is one which has remainder 0 and an odd number is one with remainder 1. For example, as $17 = 2 \times 8 + 1$ (so the remainder is 1), 17 is odd.

Examples

- How many times does 6 go into 50 and what is the remainder?
  Answer. It goes in 8 times with remainder 2 (because $50 = 6 \times 8 + 2$).

- How many times does 2 go into 30 and what is the remainder? Is 30 even or odd?
  Answer. $30 = 2 \times 15 + 0$, so 2 goes into 30 15 times with remainder 0. As the remainder is 0, 30 is even.

- Find the quotient and remainder when 23 is divided by 5.
  Answer. The quotient is 4 and the remainder is 3 because $23 = 5 \times 4 + 3$.

- This one is to make you think. How many times does 23 go into 5 and what is the remainder?
  Answer. It goes in 0 times with remainder 5 (because $5 = 23 \times 0 + 5$).

- How many times does 5 go into 35 and what is the remainder?
  Answer. $35 = 5 \times 7 + 0$, so the quotient is 7 and the remainder is 0.

Practice questions

The answers are at the end. Attempt each question before looking at the answers.

1. Find the quotient when 36 is divided by 5. Find the remainder when 36 is divided by 5.

2. How many times does 3 go into 18 and what is the remainder?

3. What is the remainder when 43 is divided by 2? Is 43 even or odd?

4. How many years and months is 43 months?

5. What is the remainder when 4657 is divided by 2^{10}?

6. How many minutes and seconds is 10,000 seconds?

7. How many hours, minutes and seconds is 10,000 seconds?

8. Find the quotient and the remainder when 49 is divided by 7.

9. What is the remainder when 58 is divided by 7?

10. What is the remainder when 7 is divided by 58?

Remainders and quotients, more examples

This page contains more examples of quotient and remainder calculations. It also contains a section on how to use your calculator to assist with these calculations.

Here is a question about remainders and quotients. It's the same question written in four different ways.

- How many times does 4 go into 14 and what is the remainder?

- How many times does 4 go into 14 and what is left over?

- What is 14 divided by 4 and what is left over?

- What is the quotient and the remainder when 14 is divided by 4?

We can picture this by taking 14 circles

○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

and putting them into groups of 4. How many groups will there be and how many circles will be left over?

○ ○ ○ ○   ○ ○ ○ ○   ○ ○ ○ ○   ○ ○

There are 3 groups of 4 and 2 circles are left over. This tells us that 4 goes into 14 3 times (because we have 3 groups) with 2 left over. We call 3 the quotient and 2 the remainder.

$$14 = 4 \times 3 + 2.$$

quotient   remainder

A calculator can be used to find the quotient and remainder. There are several ways of doing this. Here is one.

Use your calculator to find 14 divided by 4. It is 3.5 or $3\frac{1}{2}$. So the answer is a bit more than 3. That is, the answer is 3 (the quotient) and something left over (the remainder). So we know that

$$14 = 4 \times 3 + \text{remainder}.$$

A calculator can then be used to find the remainder. It is $14 - 4 \times 3$, which is 2.

Examples

- Find the quotient when 22 is divided by 4. Find the remainder when 22 is divided by 4.

  Answer: Using a calculator we find that $22 \div 4 = 5.5$ or $5\frac{1}{2}$. This means that 4 goes into 22 5 times with something left over. The quotient is 5.

  We know that $22 = 4 \times 5 + \text{remainder}$. Therefore the remainder is $22 - 4 \times 5 = 2$.

- How many times does 33 go into 1236 and what is the remainder?

  Answer: Using a calculator we find that $1236 \div 33 = 37.454545\ldots$ or $37\frac{5}{11}$. The quotient is 37. The remainder is $1236 - 33 \times 37 = 15$.

- How many times does 3 go into 12 and what is left over?

  Answer: 3 goes into 12 4 times and there is nothing left over. That is, $12 = 3 \times 4 + 0$. So the quotient is 4 and the remainder is 0.

More remainder and quotient practice questions

1. Find the quotient when 10 is divided by 4. Find the remainder when 10 is divided by 4.

2. How many times does 3 go into 11 and what is the remainder?

3. How many times does 12 go into 123 and what is left over?

4. What is 44 divided by 5 and what is left over?

5. How many times does 7 go into 24 and what is the remainder?

6. How many times does 24 go into 7 and what is the remainder?

7. What is the quotient and the remainder when 1024 is divided by 33?

Answers: The quotient is 2 and the remainder is 2. The quotient is 3 and the remainder is 2. The quotient is 10 and the remainder is 3. The quotient 8 is and the remainder is 4. The quotient 3 is and the remainder is 3. The quotient 0 is and the remainder is 7. The quotient is 31 and the remainder is 1.

Java and type `int`

Quotients and remainders can be calculated easily in Java using the operators / and %.

Division with integers in Java gives an integer answer, which means that if a and b have type `int` then $a\,/\,b$ is an integer and so might be an approximation of the number $\frac{a}{b}$. Integer division truncates to give an integer. For example, $\frac{14}{5} = 2.8$, but using the operator / with type `int` gives 2. That is, the answer given is 2.8 truncated to the integer 2.

Another operator, %, known as the *modulus* or *remainder* operator, is used with two integers and $a\,\%\,b$ returns the remainder when a is divided by b. For example, $14\,\%\,5$ is 4 because 14 divided by 5 leaves a remainder of 4.

Let's revisit some earlier examples, but using the operators % and / with type `int`.

Example 1. In the egg example with 22 eggs put into boxes of 6, the remainder (the number left over) is $22\,\%\,6 = 4$.

Example 2. How many years and months is 18 months?
There are 12 months is a year, so we divide by 12. Using type `int` we obtain $18\,/\,12 = 1$ year and $18\,\%\,12 = 6$ months.

Example 3. How many minutes and seconds is 134 seconds?
We divide by 60 because there are 60 seconds in a minute. The answer is $134\,/\,60 = 2$ minutes and $134\,\%\,60 = 14$ seconds.

Example 4. How many weeks and days is 25 days?
The answer is $25\,/\,7 = 3$ weeks and $25\,\%\,7 = 4$ days.

Many simple examples do not use negative numbers, but sometimes negative numbers are needed. In Java the number returned for $a\,\%\,b$ is $a - (a\,/\,b) \times b$. This is true for both positive and negative numbers. For example, $-14\,\%\,5 = -14 - (-14\,/\,5) \times 5 = -14 - (-2) \times 5 = -4$.

Mixed practice questions

The answers are at the end. Attempt each question before looking at the answers.

11. Find the quotient when 40 is divided by 6. Find the remainder when 40 is divided by 6. Working with integers in Java, how could the quotient and remainder be calculated?

12. How many weeks, days and hours is 4,001 hours?

13. How many times does 4 go into 14 and what is the remainder? Working with integers in Java, how could these two numbers be calculated?

14. How many times does 3 go into 16 and what is the remainder? Working with integers in Java, how could the answers be calculated?

15. Find $-9\,\%\,2$.

16. Using the operator %, how could you determine whether an integer n is even or odd? Is your answer correct when n is negative?

17. How many minutes and seconds is 200 seconds?

Answers to practice questions

1. $36 = 5 \times 7 + 1$. The remainder is 1 and the quotient is 7.

2. $18 = 3 \times 6 + 0$, so 3 goes into 18 6 times with remainder 0.

3. $43 = 2 \times 21 + 1$, so 2 goes into 43 21 times with remainder 1. As the remainder is 1, 43 is odd.

4. There are 12 months in a year, so divide by 12: $43 = 12 \times 3 + 7$, so 43 months is 3 years and 7 months.

5. $4657 = 1024 \times 4 + 561$, so the remainder is 561.

6. There are 60 seconds in a minute, so divide by 60: $10,000 = 60 \times 166 + 40$, so 10,000 seconds is 166 minutes and 40 seconds.

7. From the previous question we know that 10,000 seconds is 166 minutes and 40 seconds. There are 60 minutes in an hour, so divide 166 by 60: $166 = 60 \times 2 + 46$. This means that 166 minutes is 2 hours and 46 minutes.

   Therefore 10,000 seconds is 2 hours, 46 minutes and 40 seconds.

8. The quotient is 7 and the remainder is 0 because $49 = 7 \times 7 + 0$.

9. $58 = 7 \times 8 + 2$, so the remainder is 2.

10. $7 = 58 \times 0 + 7$, so the remainder is 7.

11. $40 = 6 \times 6 + 4$, so the quotient is 6 and the remainder is 4. In Java, the quotient is given by $40 \,/\, 6$ and the reminder by $40 \,\%\, 6$.

12. We will do days and hours first: $4,001 = 24 \times 166 + 17$, so 4,001 hours is 166 days and 17 hours.

   Now we consider the 166 days: $166 = 7 \times 23 + 5$, so 166 days is 23 weeks and 5 days.

   Putting these together we find that 4,001 hours is 23 weeks, 5 days and 17 hours.

13. In Java, the number of times that 4 goes into 14 (the quotient) is given by $14 \,/\, 4 = 3$ and the reminder by $14 \,\%\, 4 = 2$. Therefore the quotient is 3 and the remainder is 2.

14. The quotient (the number of times that 3 goes into 16) is $16 \,/\, 3 = 5$ and the remainder is $16 \,\%\, 3 = 1$.

15. Recall that $a \,\%\, b$ returns $a - (a \,/\, b) \times b$. Therefore $-9 \,\%\, 2 = -9 - (-9 \,/\, 2) \times 2 = -9 - (-4) \times 2 = -1$.

16. $n \,\%\, 2 = 0$ means that n is even. Otherwise n is odd.

   If n is negative and odd then $n \,\%\, 2 = -1$. Therefore $n \,\%\, 2 = 1$ or -1 means that n is odd.

17. There are 60 seconds in a minute, so divide by 60: $200 = 60 \times 3 + 20$, so 200 seconds is 3 minutes and 20 seconds.